A researcher wishes to calculate some statistical properties for a collection of integer data values.  The data values are represented by the array tally.  The indexes of the array represent the possible values of the actual data values from zero to the maximal value. (15 in the example below).  Each array location contains the frequency (number of occurrences) of the value corresponding to its index.  In the example below, the number contained at index 4 is 10, which means the value 4 occurs ten times in the collection of data; whereas the number contained at index 8 is 0, which means that 8 does not occur anywhere in the data collection.

| **Value** | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **Frequency** | 0 | 0 | 10 | 5 | 10 | 0 | 7 | 1 | 0 | 6 | 0 | 10 | 3 | 0 | 0 | 1 |

The data collection is being stored in the class below:

class dataCollection {

       private ArrayList tally;

        public dataCollection(){  //implementation not shown }

        //constructor - initializes data collection

        public ArrayList CalculateModes() {

            //You will write this method in part a

        }

        public int kthDataValue(int k) {

            //You will write this method in  part b

        }

        public int FindMax() {

            //implementation not shown

        }

        //other methods not shown

}

a.  You will write the function CalculateModes, which is described as follows.  CalculateModes returns an array containing the mode(s) found in the ArrayList tally.  The length of the returned ArrayList is equal to the number of modes.

A mode is defined as a value that occurs with maximal frequency.  If there is more than one such value, each is considered a mode of the data.  In the example above, the modes are 2, 4, and 11, because they each occur 10 times and all other values occur fewer than 10 times.

The function, FindMax, is available for your use.  It returns the maximum value in the data collection.  Using the example above, FindMax() returns 10.

**DO NOT WRITE THE BODY OF FindMax.**

In writing CalculateModes, you may call FindMax as specified above.

Complete function CalculateModes below.

   /\*\*

          \*This function returns an ArrayList that contains the mode(s);

     \*the list's size is equal to the number of modes

     \*/

     public ArrayList CalculateModes() {

b.  You will write the function kthDataValue, which is described as follows.  kthDataValue returns the kth data value when the data values are considered in sorted order.  Recall that the indexes of the array represent possible data values and that each array location contains the frequency of the value corresponding to its index.

In the example reprinted below, the first ten data values are 2, the next five data values are 3, and the next ten data values are 4.  kthDataValue(1) returns 2, kthDataValue(14) returns 3, kthDataValue(15) returns 3, and kthDataValue(16) returns 4.

| **Value** | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **Frequency** | 0 | 0 | 10 | 5 | 10 | 0 | 7 | 1 | 0 | 6 | 0 | 10 | 3 | 0 | 0 | 1 |

Complete the function kthDataValue below.

     /\*\*

      \* This function returns the data value that is in the kth

      \*    position within the data set.

      \*/

      public int kthDataValue(int k)